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*Indicators of Real Convergence and their Applications*

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# INDICATORS OF REAL CONVERGENCE AND THEIR APPLICATION<sup>\*)</sup>

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*Measurable aspects of the economic convergence of EU countries form the main topic of this paper. For this purpose, statistical and econometric methods are presented and applied for revealing characteristic elements of such a process. A first group of methods refers mainly to aspects such as: homogeneity, polarisation or EU countries' concentration by GDP per capita at different stages. A second group of methods refers to the intensity of several correlated processes that could ensure favourable conditions for achieving economic convergence such as: initial stage of development, evolutions in time, EU's general development, territorial cooperation.*

*Romania's situation in the context of EU countries' convergence is the prevailing topic.*

*Key words: dispersion, variation coefficient, concentration coefficient, Lorentz curve, regression model,  $\beta$ -coefficient, integrated and co-integrated series, matrix of transition probabilities, territorial interdependence, territorial index, growth index, territorial econometrics.*

*JEL: O47; C21; C22; C43; C53*

## 1. Statistical Methods for Measuring the Convergence Stage

The levels attained over time by indicators regarding homogeneity, concentration, polarisation, entropy and complementarity are relevant for assessing the extent to which convergence can be confirmed. Each of these aspects, represented by specific indicators, confirms or rejects a characteristic of convergence or a feature that determines such a process.

### 1.1. The Stage of Convergence and Dispersion Indicators.

#### $\sigma$ -Convergence

Dispersion, from the perspective of the deviation of values from a central level, is measured for an assembly of elements (countries, regions) by means of simple indicators (amplitude, deviation), but especially by means of synthetical indicators such as dispersion ( $\sigma^2$ ), square average deviation ( $\sigma$ ), variation coefficient (CV), linear average deviation ( $\bar{d}$ ). Each of these indicators expresses,

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in a concentrated form (by a number), to what extent the levels of variable  $x$  (GDP, national income, export), recorded for each entity (country, region) of an established assembly (EU), deviate from the average level ( $\bar{x}$ ). Therefore, we estimate the convergence stage at a certain time by means of an indicator of variation which actually refers to the contrary of convergence, that is, it expresses numerically how far the entities of the assembly are from that central level to which the values of the analysed indicator are supposed to converge.

If compared in time, the dispersion (irrespective of the synthetic indicators in which it is expressed), as long as its level is decreasing, allows us to consider that the convergence process becomes ever more significant.

Among the synthetic indicators of dispersion, the variation coefficient ( $CV$ ) is especially useful for comparative analyses (Dalgaard C.J., Vastrup J., 2001).

The indicator known as dispersion ( $\sigma^2$ ) results in the following way:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (1)$$

where:  $x_i$  – analysed variable (GDP per capita);

$i = 1, \dots, n$  countries (regions);

$$\bar{x} - \text{arithmetic mean } \bar{x} = \frac{\sum x_i}{n} .$$

The weighted variant of dispersion is recommended for cases when a certain level  $x$  is recurrent (occurs as a frequency  $f_i$ ):

$$\sigma^2 = \frac{1}{\sum_i f_i} \sum_i (x_i - \bar{x})^2 f_i \quad (2)$$

The equation expresses numerically the degree of dispersion of values from their mean and is sensitive to significant deviations, on one hand, and to the size order in which the values  $x_i$  are represented.

If the data refer to a sample of cases, then we get an estimated value of dispersion ( $s^2$ ) and it is advisable that the denominator of relation (1) be represented by the degrees of freedom ( $n-1$ ).

Then, we can compute, on the basis of the dispersion and the mean, the following indicators:

– square mean deviation:

$$\sigma = \sqrt{\sigma^2} \quad (3)$$

– variation coefficient of  $\sigma$ -convergence:

$$CV = \frac{\sigma}{\bar{x}} \quad (4)$$

The indicator called square mean deviation (3) represents the square mean of the deviations of values from the arithmetic mean. Therefore, if we write

$$(x_i - \bar{x}) = a, \text{ relation (3) becomes } \sigma = \sqrt{\frac{\sum a^2}{n}} \text{ (the formula of square mean).}$$

This indicator is very important for standardizing the values of variable  $x_i$ .

A reduction in the course of time in the values obtained for the square mean deviation reveals a convergence in relation to variable  $x$ . This aspect is defined as  $\sigma$ -convergence. The variation coefficient (4) expresses, in a comparable form, the dispersion in relation to the mean. The fact that its level does not depend on the indicators (specific to the assembly of entities for which they are computed) recommends this indicator for analysing the convergence (T. Villaverde Castro, 2004).

We also assume the following weighted variant of computation:

$$CV_2 = \frac{\sqrt{\sigma_{(2)}^2}}{\bar{y}} \quad (5)$$

$$\text{where: } \left\{ \begin{array}{l} \sigma_{(2)}^2 = \frac{1}{n} \sum (x - \bar{x})^2 (p_i - p) \\ p_i = \text{weight of a country's population in all countries analysed} \\ p = 1 \end{array} \right. \quad (5a)$$

A less used indicator, which has the advantage of being less sensitive to deviations above average, is the linear mean deviation:

$$d = \frac{\sum |x_i - \bar{x}|}{n} \quad (6)$$

If we agree that the key element in the relation for computing the dispersion as well as all the other derived synthetic indicators (2), (3), (4), is the difference  $(x_i - \bar{x})$ , then, the Eurostat indicator “GDP per capita in PPP EU-25 = 100” is of special importance for estimating the convergence degree. This because – while the published levels represent the proportion of value  $x_i$  (GDP per capita) achieved by country  $i$  in relation to the EU average – it allows the following:

- the analysis of the indicator evolution in time and, implicitly, its approximation to the EU average (the analysis of the dynamic series);
- the account of the countries' positioning from the perspective of variable  $x$  in relation to the mean, in a certain year, by computing the dispersion.

Therefore, if we write:

$$z_{it} = \frac{x_{it}}{x_t} \quad (7)$$

the proportion of the level achieved by country  $i$  in year  $t$  in relation to the average level, then for:

$$\frac{\sum_i z_{it}}{n} = 1 \quad (8)$$

the dispersion is as follows:

$$\sigma_{z(t)}^2 = \frac{\sum (z_{it} - 1)^2}{n} \quad (9)$$

If in the course of time the level of the dispersion computed as above undergoes a continuous decrease, we may say that the countries' convergence from the perspective of variable  $x$  develops over the entire interval.

If we confine ourselves to analysing the evolution of indicator  $z_{it}$  in each country over successive periods, we notice (concerning the convergence) in the graph a lowering slope for countries initially situated above average and a rising slope for countries initially situated below average; the slope is more or less steep in accordance with the speed at which each county is coming closer to the average that continuously change every year.

*$\sigma$ -convergence and the dynamics of dispersion of the GDP values in EU countries*

The data used for computing the dispersion indicators,  $\sigma^2$ ,  $\sigma$ ,  $CV$ , refer to the GDP per capita (PPS) and are expressed as proportions in the EU average in 2000 and 2006. The results are presented in Table 1.

Table 1

		$\sigma^2$	$\sigma$	$CV$
EU-25	in 2000	0.182146	0.426785	0.426785
	in 2006	0.190336	0.436275	0.436275
EU-27	in 2000	0.201233	0.44859	0.44859
	in 2006	0.206046	0.453922	0.453922

Both in 2000 and in 2006, EU countries was not quite a homogenous system ( $CV \leq 0.4$ ) as regards the GDP per capita position in relation to the average. In the analysed period, the dispersion was rather large so that convergence could not be confirmed in relation to the dispersion indicators.

According to the weighted variant (5a), that is, including the weights concerning the population of each country in all population of EU-27 in the years which mark a longer interval (1997–2005), the following values of weighted dispersion resulted: 0.18732 in 1997 and 0.18835 in 2005. Therefore, the dispersion degree of the GDP per capita values is the same, but with a slight decrease (at the third decimal place).

### *1.2. The Measurement of the Concentration Degree and the Analysis of Convergence*

Concentration – as increasing accumulation of achievements (incomes, immovable assets, etc.) in favour of a decreasing number of holders – expresses a state of inequality, of divergence, proportional to the degree of concentration.

In terms closer to the economic convergence field, we could say that, in a group of countries, there is a convergence process in relation to income (GDP, national income) if the proportion held by each country pertaining to the group in the number of population has a corresponding element which is getting closer in size to the proportion of that country's income in the total income of the group.

The concentration indicators show, when they are close to zero, the state of “equity”, of equilibrium or, anyhow, of proportionality between resources (population, area, number of organisations, etc.) and results (production, income, access to funding, etc.) of the components of the group.

One of the simplest procedures is the indicator proposed by Corrado Gini:

$$C_G = \sqrt{\sum_{i=1}^n g_i^2} \quad (10)$$

where  $g_i$  is the weight of the element (of the country or region  $i$ ):

$$g_i = \frac{x_i}{\sum x_i}$$

The maximum level of the coefficient is 1; the minimum level of the coefficient is not zero, but  $\sqrt{\frac{1}{n}}$ . Therefore,

$$\sqrt{\frac{1}{n}} \leq C_G \leq 1 \quad (11)$$

According to the Gini-Struck variant, the concentration coefficient is placed between 0 and 1 and, therefore, we get:

$$C_{G-S} = \sqrt{\frac{n \sum g_i^2 - 1}{n-1}} \quad (12)$$

The coefficient proposed by Herfindahl:

$$I_C = \sum_{i=1}^n g_i^2 \quad \frac{1}{n} \leq g_i \leq 1 \quad (13)$$

Each of the above coefficients indicates an increasing concentration as it is coming closer to the upper level, which is 1.

Also, the Lorenz curve refers to the question of concentration and is a usual tool of economic analysis, since it provides a picture of the process intensity, as well as a possibility for quantifying the degree of concentration.

The process involves analysing in parallel the positioning of the weights regarding two correlated variables ( $y, x$ ). The comparison of the cumulated weights computed for each entity (country, region, groups) with regard to variable  $x$  (denoted by  $g_{(xi)}$ ) can be graphically presented by placing these points in a system of coordinates:  $g_{(x1)}, g_{(y1)}; \sum_{i=1}^2 g_{(xi)}; \sum_{i=1}^2 g_{(yi)}; \dots; \sum_{i=1}^n g_{(xn)}; \sum_{i=1}^n g_{(yn)}$ . The line that links such points forms, in relation to the first bisectrix, a zone proportional by area to the degree of concentration (Figures 1 and 2).

The following relation (14) is recommended:

$$G = \frac{1}{2\bar{y}} \sum_i \sum_j p_i p_j |y_i - y_j| \quad (14)$$

where:  $p$  – weight of the population of country  $i$  in the total EU population;  
 $y$  – weight of the GDP achieved by country  $i$  in total EU GDP;  
 $\bar{y}$  – GDP per capita in the EU;  
 $0 \leq G \leq 1$ .

The Gini coefficient, based on knowledge elements provided by the Lorenz curve, is twice the area between the line that links the coordinate points given by the cumulated weights and the first bisectrix.

The concentration indexes show synthetically the actual state at a given moment with regard to the location of the weights corresponding to the units of an assembly.

Defining the indicators from a one-dimensional perspective (10, 12, 13) is justified for cases when, for equal opportunities for the units (countries, regions) within the assembly, there are unequal weights regarding the extent of the social-economic process. In the bidimensional variant, specific to the approach known as the Lorenz curve as well as the variant based on coefficient (14), we consider both the position of the weight concerning the resource (it could be population, area, number of producers in relation to the position of the weights concerning the dimension of the results (it could be GDP, income, export).

Regarding the convergence analysis, the concentration indicators provide scarce information, as they are defined to express numerically facts which are at most complementary to convergence, and this only if we use bidimensional indicators of the types of indicators inspired by Lorenz curves (14).

Of course, a comparative analysis of the concentration coefficients obtained for successive periods could indirectly signal a situation close to what we mean by convergence. This situation could be confirmed by the gradual diminution in the level of concentration towards zero. But indirectly, the concentration indicators show how far we are from the state of “equity”, from a fair proportional distribution of economic results achieved by the countries studied. Therefore, the utility of the coefficients used to measure the concentration degree for the analysis of convergence is limited. At most, the bidimensional indicators (of the Ginni coefficient type (14)) could be of some use, but only if a proportional distribution of the weights by entities (countries, regions) in relation to the couple of variables conditioned, be it only partial, the achievement of convergence within an assembly of countries. Such couples of variables in relation to which the concentration could be an indicator complementary to convergence are possibly represented by the following: population-consumption, active population-production; agricultural area-agricultural production, etc.

*The degree of concentration of EU countries in relation to the GDP per capita*

To estimate the degree of concentration and its evolution, first we grouped the EU27 countries by the level of GDP per capita in 1996 (the basis of comparison). We mention that the groups were also maintained in 2005 not only for ensuring the comparability, but especially for the fact that the homogeneity of each group did not significantly change, since only the limits of the intervals were different.

In Table 2 we specify the number of the group for which the limits of the intervals represented, in 1996, the following values (GDP per capita): below 9000 – group 1; 9001-13000 – group 2; 13001-17000 – group 3; over 17001 – group 4. The number of countries, the weights in relation to GDP and the population in 1996 and 2005 are included in the table columns.

Table 2

Group	Number of countries	Weights in 1996		Cumulated weights		Weights 2005		Cumulated weights	
		GDP	Pop.	GDP	Pop.	GDP	Pop.	GDP	Pop.
1	2	3	4	5	6	7	8	9	10
1	8	7.6	19	7.6	19	7.6	18.6	7.6	18.6
2	3	3.5	5	11.1	29	3.37	4.7	10.97	23.3
3	6	11.1	12	22.6	36	11.3	13.2	22.27	36.5
4	10	77.4	64	100	100	77.73	63.5	100	100

On the basis of the data presented in columns 5-6 and 9-10, respectively, on the cumulated weights for each reference year, we traced the Lorenz curves (Figures 1 and 2):

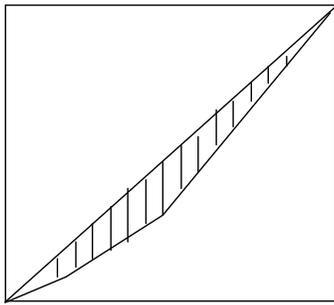


Fig 1. Lorenz Curve, 1996

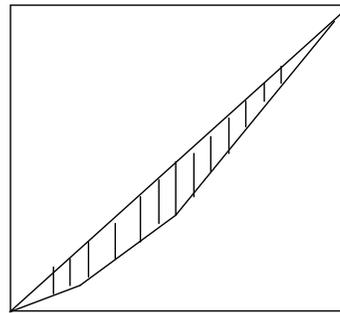


Fig. 2. Lorenz Curve, 2005

The graphs show a moderate level of concentration if we consider population a reference criterion for assessing the income (GDP). Inequalities are rather revealed by extreme weights, that is, almost one-fifth of the population (exactly 19%) produces only 8% of GDP, which is “traded off” by the fact that 64% of the EU population produces 77% of GDP.

The Gini coefficient in the weighted variant (4) was maintained at the same level if computed for the 1996 grouping of the countries,  $G = 0.155$ , as well as for the same grouping in case of increases, especially in relation to the GDP in 2005, when the level obtained was approximately the same,  $G = 0.1553$ . It results that, from the perspective of both methods for the concentration estimation, no significant changes took place, and the advance towards the state of “equity” compatible with the convergence within the EU could not be revealed, at least for the 1996-2005 interval.

### 1.3. The Theil Index and the Convergence

The convergence process taking place in an assembly of entities (countries, regions) has a corresponding notion in physics, that is, entropy which tends to permanently increase (therefore, it converges to a state of full equipartition corresponding to the second law of thermodynamics). The measurement of the degree of convergence can consequently be solved by considering the procedure of providing a quantitative measurement of the degree of non-determination (entropy).

The development level, synthetically presented through the GDP per capita in each country of an assembly (EU), is assimilated to the row of independent achievements of an aleatory dimension  $y_i$ . The additivity of entropy (one of the properties of its quantitative expression) as well as the fact that the basis of (decimal) logarithms determines the standard unit of entropy are elements on which the Theil index is defined as follows:

$$T = \frac{1}{n} \sum_{i=1}^n \frac{x_i}{\bar{x}} \log \frac{x_i}{\bar{x}} \quad (15)$$

The variant (Dickey, 2001) proposed for the case when there are several groups of countries that form the assembly is expressed by the equality:

$$T_{(d)} = \sum_{k=1}^k s_k T_k + \sum_{k=1}^k s_k \log \frac{x_k}{\bar{x}} \quad (16)$$

where:  $T_k$  – Theil index computed for group  $k$ ;  
 $s_k$  – the share in variable  $x$  (GDP per capita) pertaining to group  $k$  in all groups.

It is assumed that, as the level of index  $T$  is decreasing and coming closer to zero, the degree of divergence (as gaps, and as diversity with regard to the development level) decreases, so that we could consider that countries converge to closer values of variable  $x$ .

As regards the advantages, the Theil index expresses through a synthetical dimension, a state of the system which can be correlated with the development of a process of convergence. Moreover, it enables us to estimate “the divergence” (level distinction) also for an assembly structured by groups of entities (countries).

As for limits, we could consider the aspects suggested by the following question:

- Can the assembly of countries be considered a closed system?
- Are the countries “forming” the assembly fully economically independent?
- Can the convergence process lead to a “depleted” system, in which the transfer of “energy” is no longer possible?

### *Dealing with convergence from the perspective of the Theil index*

To determine the Theil index (15), we considered data concerning the proportion of GDP per capita recorded by each EU country in relation to the EU25 average (GDP per capita PPS EU 25=100). The GDP proportions achieved by Romania and Bulgaria were also considered for our computation. The results refer to two sufficiently distant years (1997, 2000), and after applying relation (15), they became:

$$T(1997)=171.924;$$

$$T(2006)=191.8.$$

The evolution of the indicator does not confirm the development of a convergence process in the EU, at least regarding the GDP per capita. It is worth mentioning that, in relation to other discussed indicators (of dispersion or concentration), the Theil index underlines the departure from the state of “equipartition” as we are coming closer to the present.

### *1.4. Measuring the Degree of Polarisation*

Polarisation is a process taking place along with convergence which produces homogeneous sub-groups in relation to the countries that form a sub-group, but between sub-groups the differences increase significantly in time. According to Esteban and Ray (1994) [8], the concept of polarisation cannot be neglected when analysing the persistence, the increase and the diminution in economic inequality. In this respect, it is necessary to distinguish between the convergence towards a global average level and the increasingly manifest positioning of countries by sub-groups around medium local levels. The concept of polarisation may be linked to what we call “two-gear” development and consequently, the polarisation of the society and the state of inequity.

The polarisation index can be expressed in the form of a variant of defining the concentration coefficient (14):

$$PI = \sum_i^n \sum_{j=1}^m p_i^{(1+\alpha)} p_j |y_i - y_j| \quad (17)$$

where  $p_i$  – weight of the population of country  $i$  in all countries;  
 $\alpha$  – index of concentration sensitivity  $1 \leq \alpha \leq 1.6$ .

A diminution in the level of the concentration index (17) would indicate a diminution in the polarisation in all countries analysed, which would be equivalent to a process of homogenisation.

The estimation of the degree to which polarisation reaches significant levels can be made through the F test. The dispersion analysis (based on the F test) is made through the grouping method, which enables us to study the variation in a

variable in relation to different influences that caused differences between the groups of countries. The hypothesis on which the analysis is based concerns the causes of the formation of groups at such intensity that the variation (dispersion in relation to the group average) inside the groups is lower than the variation (dispersion) in the group average as against the general average.

In case of two or more groups of countries, while each group is polarized around the so-called group average ( $\bar{x}_j$ ), we compute the dispersion as follows:

$$\sigma_{x_j/\bar{x}_0}^2 = \frac{\sum (\bar{x}_j - \bar{x}_0) n_j}{k-1} \quad (18)$$

The dispersion inside the groups,  $\sigma_{x/x_j}^2$ , is found by summing up the square differences between the levels achieved by each country ( $x_{ij}$ ) and the mean of the achievements inside that group ( $\bar{x}_j$ ).

Therefore, for group  $j$ :

$$\sigma_{x_i/\bar{x}_j}^2 = \frac{\sum_{i=1}^{n_j} (x_{i1} - \bar{x}_1)^2}{n-1} \quad (19)$$

For all groups:

$$\sigma_{x_{ij}/\bar{x}_j}^2 = \frac{\sum_{i=1}^m \sum_{j=1}^k (x_{ij} - \bar{x}_j)^2}{n-k} \quad (20)$$

To value the significance of the differences between groups and, implicitly, to check that the achieved polarisation is important (significant) and not contingent, we make the F test (of the dispersion ratio).

When the polarisation reaches a certain stage, we expect that the dispersion between groups is higher in level than the dispersion inside groups.

Therefore:

$$F_{calc} = \frac{\sigma_{x_j/\bar{x}_0}^2}{\sigma_{x_{ij}/\bar{x}_j}^2} \quad (21)$$

The result is compared with the table value F, in accordance with the selected level of significance ( $\alpha=0.05$ ) and the degree of freedom corresponding to the dispersions "involved" in computing F, that is,  $k=1$ ;  $n-k$ . Here,  $n$  = total number of countries;  $k$  = number of groups into which the countries polarized.

If  $F_{computed} > F_{tabled}$ , we estimate that there is a significant degree of polarisation. In terms of factorial analysis, it is assumed that the factor/factors which caused the formation of groups (poles) had a significant effect on the resulting features (GDP per capita).

### 1.5. Multidimensional Convergence Indicators

The above-mentioned coefficients of dispersion, concentration, polarisation, etc. generally concern one aspect. In our examples, this aspect was represented by GDP per capita, whose synthesizing power is great, but in our opinion convergence is unilaterally approached, only with regard to economic growth. An extension to other fields of social development is beneficial, which involves more synthetical indicators in computations.

This approach requires solutions to the following problems: a) the selection of synthetic indicators in such a way that the computations should include the most important criteria of estimation of the social-economic state; b) the adoption of a system of aggregation so that the obtained indicator allows for analysing convergence dynamically on a multidimensional basis.

A possible solution is the introduction of the considered indicators into computation in order to obtain “the human development index (HDI)”.

The HDI aggregate introduces into computation very relevant aspects of the social-economic state concerning economic growth, education, health condition, etc. Among the representative variables of the above aspects we find: GDP per capita, the proportion of population with a certain level of education, life expectancy. The weighted mean of the standardized values of such variables represents the level of a comparable indicator, which synthesizes at least three relevant dimensions concerning the state of a country in the reference year.

The  $\sigma$ -convergence of a multidimensional indicator would show the degree of dispersion of the countries pertaining to the assembly in year  $t$ :

$$\sigma_I = \sqrt{\frac{\sum_{i=1}^n (I_i - \bar{I})^2}{n}} \quad (22)$$

respectively:

$$CV_i = \frac{\sigma_I}{\bar{I}} \quad (23)$$

where:  $I_i$  = HDI level in country  $i$ ;

$\bar{I}$  = average level of HDI in the EU.

A downward trend of coefficients (22),(23) as the reference year comes closer to the present period:

$$\sigma_{I(t)} > \sigma_{I(t+1)} > \dots > \sigma_{I(t+n)} \quad (24)$$

would show the convergence of countries in time (the last years) with regard to several aspects relevant to the social-economic state of EU countries.

Another computation procedure could be based on variation coefficients (*CV*) obtained for each analysed variable. They could exclusively refer to the three aspects mentioned above, but other variables could be added as well: the citizens' state of safety or satisfaction, the development of some superstructure elements, etc. The capacity of the variation coefficients (*CV*) of representing a comparable dimension turns the average of such coefficients computed for *j* variables concerning country *i* in year *t*:

$$CV_{i(t)} = \frac{\sum_{j=1}^m (CV)_{j,i(t)}}{m} \quad (25)$$

into a synthetic, multidimensional indicator, so that its dispersion, decreasing in time, should reveal the convergence.

#### *1.1.6. Bilateral Complementarity of the Economies of Some Countries – An Aspect of Potential Convergence*

The proposed indicator is based on the following assumptions:

- Every country has resources or achievements that might exceed its own needs, which, in general, brings on exports – while it faces shortages in other “areas”, which lead to imports or, anyhow, to concerns about finding and drawing in foreign resources.

- There is an explicit or only implied interest in finding partners for mutual completion of resources or achievements, which would require an indicator for measuring the degree of bilateral complementarity in order to initiate or develop mutual exchanges through various forms of cooperation. The fact that countries of a built assembly, such as the EU, are neighbouring states, which provide special incentives for cross-border flows of resources, merchandise, etc. turns implicitly the recognition and the measurement of the degree of bilateral complementarity into a rigorous evaluation of a “resource” able to stimulate the cooperation between partner countries.

The complementarity coefficient proposed is defined within a range between 1 (maximum degree of complementarity) and 0 (non-complementarity).

In the bilateral case (countries A and B), if we consider  $i=1, \dots, m$  characteristics (resources, groups of goods, tourist destinations, etc.) in our computation, the coefficient is defined as follows:

$$C_{(A,B)} = \frac{\sum_{i=1}^m (w_i - 1)}{-2m} \quad (26)$$

where:

$$w_i = \frac{-x_{(A)i}}{x_{(B)i}} \quad (27)$$

if the deficient of country A ( $-x_{(A)}$ ) is lower than the surplus of country B ( $x_{(B)}$ ):

$$w_i = \frac{x_{(B)i}}{-x_{(A)i}} \quad (28)$$

if the surplus of country B is below or equal to the deficient of country A:

$$|w_i| \leq 1 \quad (29)$$

A variant of the coefficient (26) takes into account the unequal importance of the considered features, which requires the introduction of weights  $g_i$  proportional in size to the degree of importance given to that characteristic:

$$C_{(A,B)} = \frac{\sum_{i=1}^m (w_i - 1)g_i}{-2 \sum_{i=1}^m g_i} \quad (30)$$

The indicator used to measure the complementarity level shows a potential of mutually advantageous cooperation between countries that are not hindered by customs policies as EU countries are.

## 2. Indicators and Methods for Measuring the Intensity of the Convergence Process

Below, we analyse convergence as regards the relations between the approximation to the state of homogeneity and conditions that could facilitate such an objection.

Developments in time and space (territorial perspective) are especially relevant for forecasts of economic convergence.

### 2.1. Convergence and Regression Analysis. $\beta$ -Convergence

It is assumed that there is a relation of dependence between the growth rate (in a given period) and the development level of that region (country). The relation of dependence can be observed in an assembly of regions (countries) which reaches, at a reference moment ( $t = 0$ ), a certain development level.

As regards the analysis of convergence, the approximation of the development levels of the countries to each other is presupposed, that is, the

assumption concerning the higher growth rates of less developed countries ("speeding up" to catch up with the developed ones) is based on statistical data as well as on the economic theory (Solow, 1956). Of course, such a process is more or less intense, depending upon the period analysed or the social-economic state specific to that period. The regression analysis is meant to cover this aspect, that is, to quantify the marginal reaction of the effect (GDP per capita) to the modification of the cause (rather of the condition expressed by the relatively low development level in the initial stage). A statistical check (*t* test) would complete the analysis.

The regression model proposed is non-linear as follows:

$$\frac{1}{T} \log \left( \frac{y_{iT}}{y_{i0}} \right) = \alpha + \beta \log y_{i0} + e_{it} \quad (i=1,2,\dots,n \text{ country}) \quad (31)$$

where:  $y_{i0}$  = development level (GDP, for example) in the basic period;

$y_{iT}$  = development level after  $T$  time units;

$\beta$  = estimated parameter of the regression slope;

$e_{it}$  = residual value.

To estimate the polarisation, we recommend the following variant:

$$\frac{1}{T} \left[ \log \frac{y_{iT}}{y_{i0}} - \log \frac{y_{At}}{y_{A0}} \right] = \alpha + \beta \left[ \log y_{i0} - \log y_{A0} + e_0 \right] \quad (32)$$

where  $y_{At}$  is a central level around which  $i$  regions of the group,  $i=1,2,\dots,n$ , are polarized.

The problem of testing the significance of the  $\beta$  estimated parameter requires the determination of the  $t_{computed}$ , as follows:

$$t_{calculated} = \frac{\beta}{\sqrt{\sigma_u^2 / \sum (y_{i0} - \bar{y}_0)^2}} \quad (33)$$

The significance is confirmed (at an assumed risk of dimension  $\alpha$ ), if the level  $t$  (tabled) for  $\alpha$  and  $n-2$  degrees of freedom is below the computed one.

The  $\beta$  coefficient expresses, in a comparable form, to what extent the average rate increases if the development level decreases by one unit. Therefore, the parameter sign is expected to be minus.

The  $\beta$  coefficient (the regression parameter that expresses the slope of the regression line) expresses numerically a relation, considered rational by economic theory (the neoclassical theory of economic growth, which deals with the connection in a reverse sense between the intensity of the convergence process and its distance from the steady state). Statistical data on the development level in different past periods provide "the raw material", which – analysed on the basis of statistical regression – produces a result that confirms or invalidates the theory.

Therefore, the approach is of an econometric nature, regarding both the objective and the computation method. The coefficient expresses a situation valid for the time interval considered and for the assembly of countries analysed. The expression of the connection is less affected by chance (it has a character of average), but it still is the result of an estimation (based on a sample of countries); therefore, the significance testing is indicated.

The  $\beta$  parameter (coefficient) expresses a potential (a “tradition”, a situation which usually repeats) of less developed countries to achieve a higher rate than the developed countries. Among the causes of this “inequality”, we mention the low initial level ( $y_0$ ), the driving effect, etc.

Such a potential, expressed by the level and sign of the estimation of  $\beta$ , has a character of average and, from country to country, it might be different. Anyhow a propensity towards convergence can be derived.

The estimated result of the  $\beta$  parameter is compatible with convergence only if the sign is minus and its level is significant (at least, as per  $t$  test).

As regards the  $\beta$  coefficient, there is a risk of representing a distorted estimation (Galton’s fallacy) (Quah, 1993). Another limitation is caused by the existence of approximately equal conditions in the countries making up the sample, with regard to population growth, saving rate, depreciation rate. The difference in condition of this kind (to which technology could be added) rather causes a process of polarisation (Chatterji, 1993).

From a methodological perspective, we consider that the unifactorial non-linear model, in which the  $\beta$  coefficient is the key element, can be extended by adding variables. Actually, the condition of being at a low level does not compulsorily involve a high growth rate, unless, at most, there is a relatively low reference basis, which initially can be exceeded quite easily, but then it becomes an increasingly difficult task. Supposing that concerted efforts are made to catch up with “leaders”, then investments (in production, education, and superstructure) should also be considered in computation. A model corresponding to such a variant becomes:

$$\frac{1}{T} \log\left(\frac{y_{iT}}{y_{i0}}\right) = \alpha + \beta \log y_{i0} + \mathcal{I}nvest. + e_i \quad (34)$$

#### *$\beta$ -convergence and the GDP evolution in EU countries*

To apply model (31), the following should be considered:

- the basic year was 1996;
- the  $y_{iT}/y_{i0}$  ratio refers to the final year 2005 ( $y_{iT}$ ) and the GDP per capita in that year for all 27 countries as well as to the basic year ( $y_{i0}$ ) in these countries.

The application of relation (31) produced the following results:

$$\frac{1}{27} \log \frac{y_{it}}{y_{i0}} = 0,41309 - 0,19176 \log y_{i0} + u \quad \left| \begin{array}{l} R^2 = 0,387 \\ F = 15,88 \\ t = 1, 2, \dots, T \\ i = 1, 2, \dots, 27 \end{array} \right. \quad (35)$$

After making the  $t$  test (Student distribution) with regard to the significance of  $\beta$  parameter = -0.19176, the resulted level was  $t = -3.9$ , which means that the parameter significantly differs from 0 (the risk is very small, 5.2 to 10000).

The model is generally significant with regard to parameters estimated in accordance with the F test.

The resulted negative sign confirms our expectations (including the economic theory).

The relatively low degree of determination of the economic growth by the starting level – a degree illustrated by  $R^2=0.387$  also signals other causes.

## 2.2. Co-integrated Series and Convergence

The concept of integrated series of  $d$  order ( $y_t \sim I(d)$ ) refers to the fact that the non-stationary time series (therefore, having a general growing/decreasing trend, more or less accentuated) becomes stationary (without a trend) after computing the differences of  $d$  order ( $d = 1, 2, \dots$ ) from successive terms. One should not omit the possibility that  $d = 0$ , which indicates the absence of the trend (stationary series). If for two time series the integration order is identical and, additionally, the series allow for a linear combination which shows an integration order ( $b < d$ ) below the analysed series, we say that the series are cointegrated, of order  $d, b$ . The denotation used is the following:  $x_t, y_t \sim CI(d, b)$ . If two cointegrated series show different growth slopes, so that the series with a lower starting level has a steeper slope, we could say that the two series indicate a convergence process. The analysis of cointegration could be extended to more than two series, even by dealing with it through combinations of series taken two by two.

As regards the economic development of countries pertaining to a community, there are situations that cause the cointegration analysis to present utility. Among them, we find: long linear economic growth when there are different rates of economic growth; being aware of the existence of a state of statistical equilibrium in the economic evolution of the countries.

We consider the time series regarding the GDP per capita variable for country (region) A, denoted with  $Z_t$ , and the same variable for country (region) B,  $X_t$ , the latter being below the GDP per capita. If  $Z_t$ , as well as  $X_t$ , shows a linear

growing trend so that the series of differences of order 1 denoted by  $\Delta Z_t = Z_t - Z_{t-1}$ ,  $\Delta X_t = X_t - X_{t-1}$  are integrated series of order 1 and there is a linear combination of the two series (for example,  $W_t = Z_t - (a + bx_t)$  and it is integrated of order 0 (has no trend), we say that the two series are cointegrated of order 1,1 and we denote it as follows:

$$Z_t, X_t \sim CI_{(1,1)} \quad (36)$$

To determine that the convergence process is under way, it is necessary that the average of the differences for a period  $t$  should be lower than the initial difference between the development levels:

$$M((Z_t - X_t)/e_t) < (Z_0 - X_0) \quad (37)$$

More exactly, among the two series of different growing trends (regarding the slope – b) there is a relation so that:

$$\lim_{T \rightarrow \infty} M((Z_t - bX_t)/e_0) = 0, \quad b < 0 \quad (38)$$

and

$$Z_t, X_t \sim CI_{(d,b)} \quad (39)$$

*The cointegration analysis applied to time series concerning the GDP per capita in EU25 and Romania*

Parallel evolutions of GDP per capita in the EU and Romania are shown in Table 3 and in Figure 3.

Table 3

GDP per capita in constant prices (PPS) – thou. euros

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007
EU-25 (X)	18.8	19.5	19.9	20.2	20.4	20.9	21.3	21.9	22.5
Romania (Y)	4.8	4.9	5.2	5.4	5.7	6.2	6.4	6.9	7.4

The differences of order 1 between successive terms of the time series shown in Table 3 represent the results obtained by deducting the level attained in year  $t$  from the level in the year  $t+1$ . The differences are shown in Table 4:

Table 4

	$\Delta X$	0.7	0.4	0.3	0.2	0.5	0.4	0.6	0.6
For EU									
For Romania	$\Delta Y$	0.1	0.3	0.2	0.3	0.5	0.2	0.5	0.5

The evolution of the indicators presented in Table 3 is shown in Figure 3:

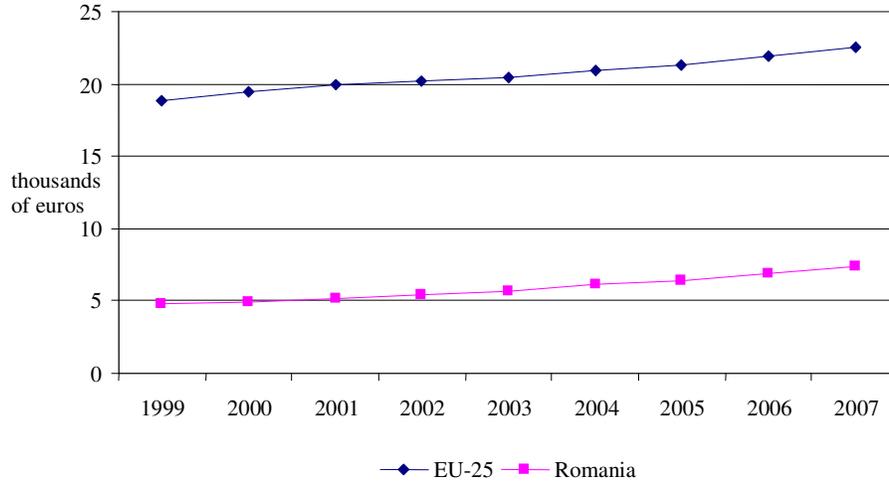


Figure 3. Evolution of the GDP per capita in the EU and Romania, 1999–2007.

Both in the EU and in Romania, the GDP per capita variable shows a linear growing trend (Figure 3). The means computed for each series concerning the GDP, while the initial year is closer to the present, become greater and greater, which confirms the existence of the trend (non-stationarity of the analysed series).

The differences of order 1 ( $\Delta X$ ,  $\Delta Y$ ) shown in Table 4 no longer present the general growing or decreasing trend, as they vary around a constant level (stationary series). Therefore, we could say that the time series  $Y_t$ ,  $X_t$  are integrated (each) by order 1.

The linear combination  $Z_t = Y_t - (a + bX_t)$  resulted from the estimation of parameters  $a$  and  $b$ , respectively from the linear model  $Y = a + bX$  could be characterized as a stationary series (integrated of order 0). Indeed, the estimation result is  $Y = -10.76148 + 0.79729 X$ , and the linear combination  $Z = Y - (-10.76148 + 0.79729 X)$  could be characterized as stationary since the obtained values:  $Z(1) = 0.5722$ ;  $Z(2) = -0.26418$ ;  $Z(3) = -0.32229$ ;  $Z(4) = -0.2804$ ;  $Z(5) = -0.11959$ ;  $Z(6) = -0.03716$ ;  $Z(7) = 0.2047$ ;  $Z(8) = 0.2466$ ;  $Z(9) = -0.2224$  have no trend.

It results that the analysed time series are cointegrated  $X_t, Y_t \approx CI_{(1,1)}$ , and both conditions (of being integrated of the same order and of allowing for a linear combination of an order lower than the integration order) are fulfilled.

Consequences: the regression analysis of such series produces good estimates of the parameters (overconsistent estimates); the evolutions described by such series are characteristic of processes in a state of statistical equilibrium. An

important aspect concerning the convergence is the “concurrence” of the slopes followed by the two linear trends. Therefore, a higher level of the parameter in its standard (comparable) form concerning the evolution starting from a low level (the case of Romania) is a condition of approximation in time to the evolution that started from a much higher level (the case of the EU).

The trends shown by the parameter estimates are described by the following equations:

– GDP per capita in the EU (constant prices):

$$y = 20.6 + 0.425t \quad (t = -4; -3; -2; -1; 0; 1; 2; 3; 4), \text{ and the comparable}$$

$$\text{(standard) level } \beta = b \frac{\sigma(t)}{\sigma(y)} = 0.425 \frac{2.582}{1.106} = 0.9928 ;$$

– GDP per capita in Romania (constant prices):

$$y = 5.87 + 0.326t \quad (t = -4; -3; -2; -1; 0; 1; 2; 3; 4), \text{ and the standardized level of}$$

$$\text{the slope } \beta = 0.326 \frac{2.582}{0.85456} = 0.9842 .$$

It results that the slopes expressed by parameter  $b$  in each equation of the trend ( $b=0.425$  in the EU;  $b=0.326$  in Romania) as well as coefficients  $\beta$  ( $\beta=0.9928$  in the EU and  $\beta=0.9842$  in Romania) are contrary to our expectations, that is, the trends described for the last nine years do not bring the economic growth convergence of our country closer to the economic growth average of the EU.

The re-computation of the trend in the last five years (2003-2007) – the period of higher growth in Romania – led to the following representations:

$$y = 21.4 + 0.52t \quad (\text{coefficient } \beta = 0.99698) \text{ in the EU;}$$

$$y = 6.54 + 0.41t \quad (\text{coefficient } \beta = 0.9966) \text{ in Romania.}$$

We notice that, when dealing with trends for a more recent period, the estimates of parameter  $b$  (slope of the regression line), as well as their standardized values ( $\beta$ ) retain the conclusion that the trends of the last decade both in the EU and Romania do not represent an argument to confirm the convergence in relation to the GDP per capita. If we consider the same context of the slopes described by trends, then keeping the marginal value over 0.45 (see parameter  $b$  computed for the last nine years), the situation changes and the approximation is more and more obvious.

### 2.3. *Convergence after Some Probable Transitions in Time. Matrix of Transition Probabilities*

The early stage consists in working out a classification by categories (classes, intervals) in relation to the level of a defining characteristic of the development level (GDP, HDI, etc.). Although, theoretically, such a distribution of countries by

groups is difficult to make, from a practical perspective such classes, groups and categories represent a procedure frequently discussed in economic literature.

An assumption, on which the process described below is based refers to the transition of several countries from a category to another (usually to a higher one). Transitions take place at an intensity that repeats every year.

Therefore, it is assumed that the proportion or, more exactly, the probability to move from category  $I$  to category  $j$  ( $i \leq j$ ) stays approximately at the same level every year. Consequently, starting from a certain distribution of countries by categories (classes), one comes gradually, in time, to another distribution. The latter can quite easily be predicted if the “style” of transitions (that is, probabilities of transition) occurring in the past is also maintained in the future.

As countries are “agglomerated” towards a certain category (usually, a higher one), we could say that a process of convergence evolves in time and, under normal conditions, its evolution and finality are predictable.

The key element is the probability of transition from category  $i$  to category  $j$  in period  $(t, t+1)$ . If each category contains a certain number of countries and if, when entering the next year, some countries migrate to other categories (the rest of them remains consistent with the category to which they pertained), then the probability of transition from  $i$  to  $j$  in  $(t, t+1)$  is derived as follows:

$$P_{ij} = \frac{x_{i \rightarrow j}^{(t,t+1)}}{x_i^{(t)}} \quad \left( \begin{array}{l} t = 1, \dots, T \\ i, j = 1, n \end{array} \right) \quad (40)$$

After computing the probabilities for  $i=1, \dots, n$  and  $j=1, \dots, n$ , ( $n$  = number of groups), we insert them in a table of  $n$  rows and  $n$  columns, that is, the formation of a matrix of transition probabilities:

$$M = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{pmatrix} \quad (41)$$

The distribution of countries by groups in the next year  $(t+1)$  takes into account the positioning of the countries by groups in year  $t$  as well as matrix  $M$ . If we denote the structure of the countries by categories (groups) with  $P_{i(T)}$ , then the future structure of the countries by groups ( $P_{j(T+1)}$ ) is the following:

$$P_{j(T+1)} = P_{j(T)} \cdot M \quad (42)$$

To forecast the distribution of countries in  $k$  years, we consider the following equalities:

$$P_{j(T+K)} = P_{j(T+K-1)} \cdot M = [P_{j(T+K-2)} \cdot M \cdot M] = P_{i(T)} \cdot M^K \quad (43)$$

The fact that the repositioning by categories in year  $T+K$  depends on the positioning in the previous period  $T+K-1$  induces the idea of chain in which each link (that is, distribution) depends on the previous state; that is why it is called a Markov process (chain).

This method produces results compatible with characteristics of the convergence process if it implies the regrouping of the countries into a smaller number of groups (possibly, a simple group).

If the countries tend to regroup into a single category during the forecasting period, we could say that the convergence process is under way; if not a single “dominant” emerges, but two or, possibly, three groups they tend to gradually include all countries of the analysed assembly, then there is rather a process of polarisation.

The fragile element of the procedure is the assumption that the transition probabilities (40) is constant for a sufficiently long period.

This method deserves re-evaluation, that is, laying a stronger stress on the polarisation of the countries, and this in cases when isolated groups (they have no common borders, or they are not “neighbours”) become increasingly attractive (that is, the number of countries increases in these groups). Besides, the periodical updating of the transition probabilities is highly required.

*Matrix of the transition probabilities and the changes in the structure by groups of EU countries*

To determine the elements of the matrix of transition probabilities (transitions from a group of countries to another) we used data on EU 25 GDP per capita = 100%. For the group of EU countries, we assume that the GDP average evolves continuously in time. Therefore, the limits of each interval of groups were centred on the average by introducing into computation the distance (rendered as proportion) from the average of each year in the 2000–2006 period. To obtain stable transition probabilities, we computed the average of the proportions that represented transitions from one group to another in year  $t$  as against the previous year (see relation 41). Therefore, the average of 6 proportions representing transitions from group  $i$  to group  $j$  in year  $t$  as against the existing number of countries in year  $t-1$ . Of course, this happens while there are transitions over the entire interval every year. In fact, the transitions were numerically reduced, which can be observed if we analyse the level of non-diagonal elements of the matrix presented below. Table 5 shows the matrix of transition probabilities, as well as of

the intervals used, whose limits express the proportion of GDP per capita in relation to the EU average.

Table 5

Group interval (%)	under 75	75-85	85-95	95-105	105-115	115-125	over 125
under 75	0.982	0.018	0	0	0	0	0
75-85	0.045	0.845	0.110	0	0	0	0
85-95	0	0	0.950	0.050	0	0	0
95-105	0	0	0	1	0	0	0
105-115	0	0	0	0.010	0.938	0.052	0
115-125	0	0	0	0	0	1	0
over 125	0	0	0	0	0	0.0468	0.9532

The number of countries grouped in 2000 as well as at the end of the interval (2006) is shown in Table 6.

Table 6

Year	under 75	75-85	85-95	95-105	105-115	115-125	over 125
2000	10	4	1	0	6	2	4
2006	10	2	2	2	4	4	3

We notice (Table 5) that a characteristic of the structural changes in time is the extremely limited character of the transitions from one group to another. It makes the evolution in the economic growth field take place by approximately equal steps, advancing “along with the group”. Nevertheless, if we accept an optimistic variant (favouring the convergence), we find a relatively slow orientation towards the average, an aspect clearly shown in Table 6.

The forecast made for the next period of six years implied the multiplication of the vector of country distribution in groups in 2006 by the matrix of transition probabilities (see relation 43). Therefore, we multiply  $(10 \ 2 \ 2 \ 2 \ 4 \ 4 \ 3)$  by matrix:

$$\begin{pmatrix} 0.982 & 0.018 & 0 & 0 & 0 & 0 & 0 \\ 0.045 & 0.845 & 0.11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.95 & 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0.938 & 0.052 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0468 & 0.9532 \end{pmatrix}^6$$

The table below shows the forecast (rounded figures) regarding the structure by groups of present EU countries in the year 2012:

Tabelul 7

	≤ 75%	75–85%	85–95%	95–105%	105+115%	115-125%	≥ 125%
Year 2012	9	1	3	3	3	6	2

The weak element of the entire procedure concerning the utilisation of the method for obtaining such forecasts is the continuation, for relatively long periods, of past behaviour (we refer to the proportion of transitions from one group to another). Results confirm (with the above reservation) the slow convergence process.

#### 2.4. Spatial Dependence Test. The Territorial Analysis of Convergence

A factor for achieving convergence could be the territory (country, region) in the vicinity of the country  $i$  ( $i=1...n$ ). In a cross-section series, the observed territorial units (countries) have values which are not independent from each other (that is, the level of country  $i$  can be independent from the level of country  $j$ , especially when they are neighbouring countries). Therefore, there is a functional dependence between the economic performance of country  $i$  and what happens in another country (Anselin, 1988).

A coefficient that would show how significant such a functional connection is could be a support (an objective argument) for accepting the hypothesis concerning the territorial global dependence.

Therefore  $\beta$ -convergence could be reformulated in terms of territorial (spatial) regression analysis.

To check the significance of territorial interdependence, we suggest the following computed value (Moran, 1992):

$$I_t = \frac{n}{S_0} \cdot \frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij} \cdot Y_{it} Y_{jt}}{\sum_{i=1}^n \sum_{j=1}^n Y_{it} Y_{jt}} \quad (44)$$

where:  $Y_{it}$  = log GDP for country  $i$  in year  $t$ ;

$w_{ij}$  – element referring to the territorial “correspondence” between country  $i$  and country  $j$ , as it appears in the matrix of correspondences, denoted by  $W$ ;

$n$  – number of countries considered;

$S_0$  – sum of all elements of matrix  $W$ .

$W$  – matrix whose elements represent the degree of territorial approximating between area  $i$  and area  $j$ . Each element represents a standard value,  $w_{ij} = \frac{\text{correspondence between } i \text{ and } j}{\text{sum of the elements in row } i}$ , so that  $\sum_j w_{ij} = 1$ .

In the simplest case, we notice the correspondence  $c_{ij}=1$  for two neighbouring countries (with common border) and  $c_{ij}=0$  for two non-neighbouring countries (no common border).

Therefore, in case we consider the countries A, B and C, when A neighbours on B, B on A and C, the elements  $c_{ij}$  of the intermediate matrix are obtained from the following table:

	A	B	C
A	0	1	0
B	1	0	1
C	0	1	0

and in accordance with the table data we get the matrix of standard values ( $W$ ),

$$W = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \quad (45)$$

Considering three countries,  $n=3$  and  $s_0 = 1 + \frac{1}{2} + \frac{1}{2} + 1 = 3$ .

Coefficient  $I$ , resulted from formula (44), is statistically proved. For this we recommend the t test ( $\alpha=0.05$  and  $n-2$  degrees of freedom). In case of significance of coefficient  $I$ , coefficient  $\beta$  can be obtained from representations based on the assumption presented below:

a) in case that the deviation (error -  $e$ ) is dependent on territorial launchings (deviation of country  $i$  in relation to neighbouring country  $j$  depends on the deviation in relation to the other country  $k$ ). Therefore, if we denote the deviation by  $e$ , then:

$$e_{ij(t)} = \rho e_{ik(t)} + u \quad (46)$$

where:  $\rho = \lambda w$ ;

$\lambda$  – territorial scalar;

$u$  – error that follows a normal distribution of zero average and constant dispersion.

In this case,

$$\frac{1}{T} \log \left( \frac{y_{iT}}{y_{i0}} \right) = \alpha + \beta \log y_{i0} + (1 - \rho) u_{it} \quad (47)$$

b) in case that the country interaction represents the factorial variable:

$$\frac{1}{T} \log \left( \frac{y_{iT}}{y_{i0}} \right) = \alpha + \beta \log y_{i0} + \left( \frac{\rho}{T} \right) \log \left( \frac{y_{iT}}{y_{i0}} \right) + e_t \quad (48)$$

c) the territorial variable is independent, so that the difference in relation to time 0 (the initial period considered) is added:

$$\frac{1}{T} \log \left( \frac{y_{iT}}{y_{i0}} \right) = \alpha + \beta \log \rho \log y_{i0} + u_{iT}, \quad (49)$$

where  $\rho = \tau\omega$ .

The territorial index (44) – as well as the entire procedure according to which the distance (in the strict sense of the word) is considered one of the causal variables in the convergence process – is important since it expands the research area by introducing spatial influences in economic research. Spatial econometrics is mainly focused on such aspects. By introducing the territorial element (the disposition throughout the countries considered) into the analysis of the income interdependence (GDP per capita), we can make a more rigorous estimation of the convergence process [10].

Since the significance of territorial variable ( $\rho$ ) is proved before introducing it into the regression analysis (47), we are right to affirm that proving the significance of territorial interdependence (Moran index (44)) is a strictly necessary stage and the procedure is worth generalizing also in case of other representations.

Among the weak elements, we find a somewhat subjective procedure of defining the elements of matrix  $W$  (45), as well as the risk of multicollinearity, since, quite frequently, the initial level of the indicator ( $y_{i0}$ ) can be strongly correlated with that of the neighbouring countries, so that keeping both explanatory variables (starting basis and territorial neighbourhood) could affect (distort) the parameters.

## 2.5. The Analysis of Convergence Based on Spatial Econometrics

Spatial econometrics deals with economic development from an interregional perspective, that is, the data refer to regions (countries) and influences are not considered in time, but in space, because of the vicinity that conditions the trade as well as the information, behaviour exchanges, etc., thus becoming a special factor of economic growth. More exactly, spatial approximation can be considered a

factor of economic growth, just how time was an equally special “factor” for computing the tendency. As regards the justification of special influence, we should consider the formulation of regional economic policies, the migration flows, the competition, etc.

The spatial econometric model show some resemblance to the “classical” one, if we consider the possibility to work it out in the form of a single equation or several equations. The equations of the model might include synchronous, lagging, dichotomic, anticipated variables. Its specificity consists of aspects concerning the cross-section approach, according to which the level of each variable is considered for spatial units and not in time, the lag is spatial, the theoretical ground could be economic, as well as historical and sociological, since the number of cases is small.

As regards the convergence, the model includes, as factors of influence, vicinity (in general, spatial but also economic distance), customs duty exemptions, spatial influence, capital flows, etc.

We present below a model of this kind, which includes among the endogenous variables the variable that expresses the distance between the GDP per capita of country  $r$  and the average GDP per capita of EU countries. The model, in an extremely aggregated form, is presented below:

$$\Delta PIB_r = a_0 + a_1 \Delta INV_r + a_2 \sum_{i=1}^{26} PIB_{i \neq r} + u_{1r} \quad (50)$$

$$\Delta INV_r = b_0 + b_1 \Delta PIB_{t-1,r} + b_2 \Delta TX_r + u_{2r} \quad (51)$$

$$\Delta CONS_r = \Delta PIB_r - \Delta INV_r \quad (52)$$

where:  $\Delta$  – distance in relation to EU average of GDP per capita, investments ( $INV$ ), taxes ( $TX$ ), consumption (global demand) ( $CONS$ ).

## 2.6. Unequal Indexes of Economic Growth and Convergence

The first argument for using indexes takes into account the unequal growth rates of the countries (regions) pertaining to a territorial assembly. Inequality seems to be caused by the reference basis (GDP level in the reference year) which usually differs, so that a low reference level offers an opportunity for a higher growth in time.

Another argument implies the interest in economic development and, consequently, stronger efforts towards investment (level and efficiency alike) made by less developed countries pertaining to the analysed territorial assembly. If a high growth index of a “backward” country – in comparison with the other ones – is maintained for several years, this leads to the gap filling and convergence achievement.

The growth index results from a ratio of the correct level ( $t$ ) and the reference level ( $I$ ):

$$I_{t/I} = \frac{y_t}{y_I} \quad (53)$$

and, if the basis is mobile (in a chain),

$$\bar{I} = {}_{T-j}\sqrt{\frac{y_2}{y_1} \cdot \frac{y_3}{y_2} \dots \frac{y_T}{y_{T-1}}} = {}_{T-j}\sqrt{\frac{y_T}{y_1}} \quad (54)$$

Therefore, the average growth rate is the following:

$$\bar{R} = \bar{I} \cdot 100 - 100 \quad (55)$$

If we agree on the assumption that the index is maintained in the future at a level close to the average, then, in the next period ( $T+1$ ), the GDP level (forecast) is the following:

$$y_{T+1}^* = y_1 \bar{I}^{(T)} \quad (56)$$

and, for future periods  $j$ :

$$y_{T+j}^* = y_1 \bar{I}^{(T+j-1)} \quad (j = 1, 2, \dots, J) \quad (57)$$

If another more advanced area is the target or if the average of growth rates in the countries of the assembly ( $\bar{I}^{(A)}$ ), which could be the EU, is considered, then the achievement of convergence (as GDP equalisation) implies the equality:

$$y_R \bar{I}_R^T = y_A \cdot \bar{I}_{(A)}^{(T)} \quad (58)$$

where  $\begin{cases} \bar{I}_R > \bar{I}_A; \\ T = \text{the year when country R reaches the level of country A.} \end{cases}$

or

$$\log y_R + T \log \bar{I}_R = \log y_{(A)} + T \log \bar{I}_{(A)} \quad (59)$$

that is:

$$T \log \bar{I}_R - T \log \bar{I}_{(A)} = \log y_A - \log y_R \quad (60)$$

$$T = \frac{\log y_A - \log y_R}{\log \bar{I}_R - \log \bar{I}_A} \quad (61)$$

If the objective is the equalisation time and this time is preset, we need to find the level of the growth index (average) that has to be maintained throughout the period  $t=1,\dots,T$ , so that the objective should be attained:

$$\log \bar{I}_R = \frac{\log y_A - \log y_R}{T} + \log \bar{I}_{(A)} \quad (62)$$

This is one of the simplest methods as regards the application and is based on an optimistic variant, according to which neither important accidental events nor pace changes (with both “partners”) take place in the future. Anyhow, this procedure should be considered for the analysis of convergence, be it under the form of a working variant.

For a more rigorous approach, we could relax the conditions that determine the prognosis (58), that is, by introducing into computation some “pessimistic” variants for the future growth rate average, which leads to a prognosis period with better chances to be confirmed by reality.

#### *Average indexes of economic growth and requirements for achieving convergence*

Further, we try to set the forecasting horizon at which Romania can achieve, under certain conditions, a significant approximation to the EU GDP per capita average. For this purpose we take into account different growth indexes recorded between 1999 and 2007, for which we computed average indexes, based on annual growth rates of the GDP expressed in constant prices. The hypotheses considered for Romania’s economic growth and initial level are among the most optimistic ones. Thus, the levels considered are the following:

– average growth index for the EU,  $\bar{I}_{UE}=1.029$ ;

– average growth index for Romania,  $\bar{I}_R=1.0559$ .

The initial levels are based on values recorded in 2005, and the growth rate in 2006 and 2007. The obtained levels are the following:

– for the EU,  $y_{UE}=24.8$ ;

– for Romania,  $y_R=9.3$ .

Moment  $T$ , for achieving the equality (given by relation 58), is the following:

$$T = \frac{\log y_{UE} - \log y_R}{\log \bar{I}_R - \log \bar{I}_{UE}} = \frac{\log 24,8 - \log 9,3}{\log 1,0559 - \log 1,029}$$

At the result is far enough in time (about 38 years), any comments seem to be of low interest, to say nothing about the large number of unknown quantities that might occur in such a long period. In our opinion, it is advisable to determine the average growth index, which would bring our country’s GDP per capita at the EU average level in a more reasonable time, let’s say, 15 years (relation 59):

$$\log \bar{I}_R = \frac{\log y_{UE} - \log y_R}{15} + \log \bar{I}_{UE} = \frac{1.39445 - 0.96848}{15} + 0.01242 \approx 1.1$$

Therefore, an annual growth around 10%, while maintaining the growth at an EU average of 3%, would determine a relatively quick convergence, which assumption is not quite probable.

### 3. Conclusions

The topic of the paper, focused on measurable aspects of the convergence with EU countries, is approached by several statistical and econometric methods in order to obtain useful indicators for describing the process and for analysing and forecasting its evolution.

The multitude of aspects concerning the convergence such as homogeneity of economic and social performance, concentration and polarisation, different levels and rates of development, dependence on certain circumstances, effects of driving and complementarity in the economy, etc. requires several methods and representations (models), so that the information resulting in indicators should enable us to understand the process in detail.

As regards the methodology for obtaining indicators corresponding to convergence, we point out that the methods used allow for quantifications of relevant aspects concerning the process condition, although the indicators resulted from a static perspective (for a given period). It is necessary to repeat calculations for several successive periods in order to assess the convergence without excluding possible divergences. Especially, the first group of methods (see paragraph 1) is illustrative of this case. A basic recommendation for the above methods is the use of comparable measures (weights, standard values). The fact that the resulted indicators (variation coefficient, Gini coefficient, Theil index, polarisation index) refer to a single variable (in our case, GDP per capita) is a limitation. In our opinion, it is advisable to expand the coverage area of the indicator and include into calculus several economic as well as social variables, so that the findings should be referred to a convergence on several planes. In paragraphs 1.6. (multidimensional indicators) and 1.7. (bilateral complementarity) we suggested indicators of this kind. The introduction of conditions and factors that determine such a process into calculations requires quantifications of the driving effect, the cooperation potential, the role of territorial proximity. In this respect, we present methods concerning co-integration, complementarity, and spatial econometric modelling. Many of the recommended methods are characterized by certain constraints and requirements such as: steady development rates, no major disturbing factors, repeatable transitions (from one development stage to another). Such requirements could be accepted for analyses and forecasts as intermediate objectives for achieving convergence.

As regards the stage of convergence of the EU countries, there is an alarming situation if things are considered from the perspective of less developed countries. And it happens because convergence seems to be contradictory: by the variation coefficient and the Theil index, the process is rather divergent, and by the Gini coefficient, the concentration level is maintained in time (the periods considered cover the last 7–10 years); the transition of the countries to the medial interval cannot be easily observed (the Markov process); the dependence of the growth rate on the starting level is confirmed ( $\beta$ -convergence), considering a low determination. Therefore, at least regarding the GDP per capita in the last ten years, EU countries' convergence gives very weak signals of being successful. An explanation could be two contradictory trends: convergence of the economies of old members, and divergence caused by new EU members, whose development levels are rather low ranked than middle-ranked.

As regards Romania, the co-integration analysis shows rather a departure from the EU development average, considering the growth rate existing between 1999 and 2007. From an optimistic perspective, that is, an approximation to the last years' rates (both in Romania and in the EU), the convergence could be confirmed but on quite a far forecasting horizon. According to the variant implying a steady economic growth of Romania of about 10%, a significant approach to the EU average could be possible in the next 15–20 years.

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